$$(\tfrac{1}{2}b\cos A-\tfrac{1}{2}d) : \frac{b-d}{2}=\frac{b+d}{2} : x.$$

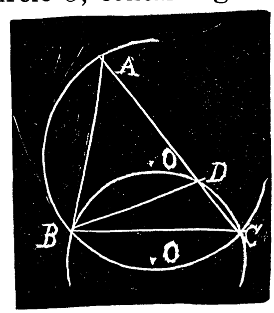
Construction. Draw AC equal to b, draw the indefinite line AW, making an angle at A equal to the given angle. On AW lay off AD equal to $\frac{1}{2}b$, and draw DE perpendicular to AC. From E lay off EI equal to $-\frac{1}{2}d$. Then will AI equal $\frac{1}{2}b\cos A-\frac{1}{2}d$.

On AW lay off AH equal to $\dfrac{b-d}{2}$ and draw HI. On AC lay off AM equal to $\dfrac{b+d}{2}$, and through M draw a line parallel to HI meeting AW at B. Join BC. Then will ABC be the triangle required.

Similarly solved by G. B. M. Zerr.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

On the given base, BC, as a chord, describe a circle O, containing the segment whose angle contains the angle$=90°+\frac{1}{2}A$, A being the given vertical angle, and also a circle O', the segment of which contains the angle A. Make BD=given difference of sides; extend BD to A, where it cuts the circumference of circle O'. Draw AC; then ABC is the required triangle.



For, $\angle ADC=90°-\frac{1}{2}A$.

$\therefore \angle ACD=90°-\frac{1}{2}A$; $\therefore AD=AC$; $\therefore AB-AC=BD$=given difference, which proves construction.

Solved similarly C. N. Schmall and H. C. Feemster.

357. Proposed by E. R. HOYT, St. Louis, Mo.

A room is 30 feet long, 12 feet wide, and 12 feet high. At one end of the room, 3 feet from the floor, and midway from the sides, is a spider. At the other end, 9 feet from the floor, and midway from the sides, is a fly. Determine the shortest path by way of the floor, ends, sides, and ceiling, the spider can take to capture the fly.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

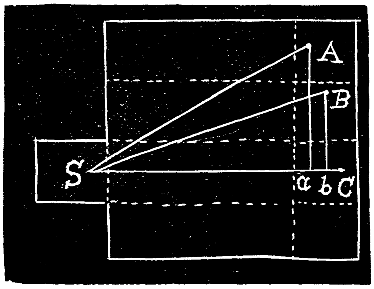
Suppose the six sides of the room spread out in one plane as in the figure, the floor being the second rectangle from the bottom, and let x=distance of spider from floor, $12-x$ the distance of the fly, $x<6$. There are three courses for the spider to take.

First, the route $SC=30+x+12-x=42$ feet...(1).

Second, the route $SB=\sqrt{[(Sb)^2+(Bb)^2]}=\sqrt{[(36+x)^2+(18-x)^2]}$...(2).

Third, the route $SA=\sqrt{[(Sa)^2+(Aa)^2]}=\sqrt{[(30+2x)^2+(24)^2]}$...(3).

Let $(30+2x)^2+576=(36+x)^2+(18-x)^2$.



Then $x^2+42x=72$, and $x=1.6495$ feet.

Let $(36+x)^2+(18-x)^2=(42)^2$.

Then $x^2+18x=72$, and $x=3.3693$ feet.

Then if $x<1.6495$ feet, the third route is shortest.

If $x>1.6495$ and <3.3693, the second route is shortest.

If $x>3.3693$, the first route is the shortest.

Since $x=3$, the spider takes the second route, and travels $\sqrt{[(39)^2+(15)^2]}=3\sqrt{[13^2+5^2)}=3\sqrt{(194)}<[3\sqrt{(196)}=3\times14=42]$ feet.

Also solved by C. N. Schmall.

358. Proposed by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Neb.

Cut four coplanar non-copunctual straight lines in a harmonic range.

I. Solution by the PROPOSER.

Let a. b, c, and d be the four lines, not meeting in a point. Let a and b meet at x. Draw two lines cutting a, b, and d at A, A_1, A_2 and A, B_1, B_2, respectively. Draw A_1B_2 and A_2B_1 meeting at C_1. Draw AC_1 cutting b and d at C and C_2. Draw XC_2, and draw XC_1 cutting C at D_2. Draw C_2D_2 cutting a and b at D and D_1. $DD_1D_2C_2$ is the required range.

For ACC_1C_2 is a harmonic range determined by the four-point $A_1B_1B_2A_2$. Hence, $X-ACC_1C_2$ is a harmonic pencil. So also is $X-DD_1D_2C$; therefore DD_1D_2C is the required harmonic range.

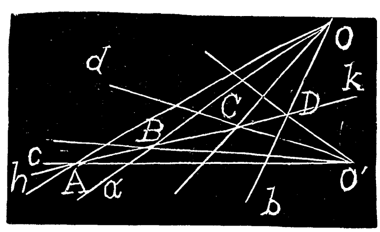
Also solved by C. N. Schmall.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let Oa, Ob, $O'c$, $O'd$ be the four lines; hk the transversal intersecting $O'c$, Oa, $O'd$, Ob in A, B, C, D, respectively.

Let $\alpha=0$ be the equation to Oa; $\beta=0$ the equation to Ob; $\gamma=0$ the equation to $O'c$; $l\alpha-m\beta+n\gamma=0$, the equation to $O'd$.

Then for a harmonic range,



$\alpha-p\beta=0$ is the equation to OC;
$\alpha+p\beta=0$ is the equation to OA;
$\gamma-q(l\alpha-m\beta+n\gamma)=0$ is the equation to $O'B$;
$\gamma+q(l\alpha-m\beta+n\gamma)=0$ is the equation to $O'D$.

$\left\{-\frac{2p\Delta}{b-ap}, \frac{2\Delta}{b-ap}, 0\right\}$,. are the coordinates of A;